Engineering Notes

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Maximum Noise Abatement Trajectories

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I. Introduction

THE takeoff and climbout maneuver of a STOL aircraft (a jet powered augmentor-wing vehicle) can produce noise of great annoyance to the surrounding communities. The object here is to determine flight profiles which will minimize that annoyance. The perceived annoyance is described by discrete measures of instantaneous noise as a function of engine thrust and distance from the noise source, and by weighting these noise measures according to the population density, the sensitivity of the population to its noise environment, and the duration of the noise.[1] This annoyance function considers a finite number of listeners located at intervals along the extended runway centerline.

Previously, dynamic programming was applied[2,3] because it allowed inequality constraints in the control and/or state to be included easily. However, the dynamic programming approach limited the number of state variables to two and the number of listeners to only one. In contrast, by using a steepest descent optimization procedure,[4] a particular STOL aircraft and the environment can be described in more detail. That is, the present program contains four state variables with ten listeners for the two dimensional problem considered and five control variables are modulated within given bounds. The performance index is formulated such that it explicitly assigns the same relative importance to thrust as it does to distances between discrete listeners and the aircraft. However, using a steepest descent optimization program, results indicate that it is preferable to keep the thrusters at their maximum value to minimize the integrated annoyance rather than to reduce thrust which would lower the instantaneous annoyance. Thrust decreases below its bound only when the instantaneous noise at a listener is limited. Furthermore, from various initial conditions, the vehicle settles quickly into a steady state of maximum sustainable flight path angle; that is, the rate of change of altitude with range is a maximum.

II. Formulation

The functional form of the performance index is

$$PI = \sum_{i=1}^{n} P_i S_i \left[\int_{t_i}^{t_i+\Delta t_i} \phi_i \, dt \right] \tag{1}$$

where i represents a particular listener for a total of n. The integral $\int \phi_i \, dt$ is the perceived annoyance, P_i is population density, S_i is the sensitivity of the population to its noise environment, and Δt_i is the interval during which the noise is considered annoying.‡ The above performance index is based on the physiological hypothesis that an annoyance function is perceived through a time integration of the total noise environment.

The form of the perceived annoyance[5] is

$$\int_{t_i}^{t_i+\Delta t_i} \phi_i \, dt = K \log_{10} \left\{ \frac{1}{t_o} \int_{t_i}^{t_i+\Delta t_i} 10^{PNL_i/K} \right\} dt \tag{2}$$

where PNL_i is Perceived Noise Level (PNdb); t_o is the time constant used to normalize the duration of the instantaneous PNL ($t_o = 10$ sec); K is a constant determining duration penalty ($K = 10$); and PNL_i is a function of thrust, T, and distance from aircraft to listener, Y_i, given as

$$PNL_i(T, Y_i) = PNL(Y_o) + \Delta PNL(T) + \Delta PNL(Y_i, T) \tag{3}$$

$PNL(Y_O)$ is the reference noise level at a 200 ft radius where the weight flow through the fan and the tip speed of the fan blade are assumed for maximum thrust. The value of 114 PNdb is chosen to represent a typical turbofan design.[1] $\Delta PNL(T)$, the reduction in fan noise due to reduced thrust, is chosen as $25 \log_{10} (T/T_{MAX})^6$. T_{MAX} is maximum sea-level thrust. This expression is reasonable for a turbofan design with a moderate by-pass ratio i.e., 1/6. $\Delta PNL(Y_i, T)$, the reduction of PNdb as a function of distance[6] is given as

$$\Delta PNL(Y_i T) = -\left[\alpha \log_{10} \frac{Y_i}{Y_o} + \beta Y_i \left(\frac{Y_i}{Y_o} - 1 \right) \right] \tag{4}$$

where α is chosen as 21.1056 to represent in flight fan noise. β is proportional to the atmospheric humidity absorption coefficient and, therefore, is a function of the fundamental frequency, f, of the fan. For a representative turbofan engine, $\beta = 1.069 \times 10^{-3} + 8.6148 \times 10^{-7} f$. At full thrust f is chosen as $f_o = 2500$ Hz. At less than full thrust f is reduced to $f_o (T/T_{MAX})^{1/3}$ (Ref. 6).

The assumptions are: 1) that fan noise dominates jet noise,[7] 2) that fan noise is not highly directional i.e., a spherical radiation model is used,[8] and 3) that there are no tonal concentrations. Given that noise field prediction techniques are not well developed, the noise model presented above is felt to be a reasonable representation of the noise sources.

1. *Equations of Motion*

The state variables are altitude, h, downrange, R, velocity magnitude, V, and flight path angle, γ. The control variables are angle of attack, α, angle of incidence (jet engine nozzle deflection), i, primary thrust, T_p, augmentor trust, T_a§, and flap angle, δ_F (Fig. 1). The primary thrust

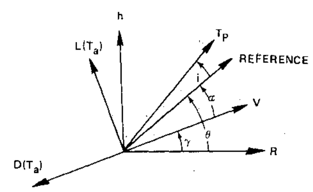

Fig. 1 Description of state and control variables.

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‡The interval is taken as the time interval of the problem for convenience in the following numerical results.

§The augmentor thrust, T_a, does not appear explicitly in the equations of motion, but is included in the lift and drag coefficients.

and c augmentor thrust are assumed separately controllable. The thrust T is the sum of T_p and T_a.

The equations of motion of a point mass representation of the vehicle are

$$\dot{h} = V \sin\gamma \,;\; \dot{R} = V \cos\gamma \tag{5}$$

$$\dot{V} = qg/(W/S)[C_J \cos(\alpha + i) - C_D] - g \sin\gamma \tag{6}$$

$$\dot{\gamma} = qg/(VW/S)[C_J \sin(\alpha + i) + C_L] + \left[\frac{V}{R+h} - \frac{g}{V}\cos\gamma\right] \tag{7}$$

where

$$C_J = T_p/qS;\; C_{J_a} = T_a/qS \tag{8}$$

$$C_L = C_L(\alpha, C_{J_a}, \delta_F)\,;\; C_D = C_D(\alpha, C_{J_a}, \delta_F)\,;\; q = 1/2\rho V^2 \tag{9}$$

The air density is ρ, the wing loading of the aircraft is W/S. Aerodynamic lift and drag are L and D, respectively. The lift and drag coefficients, C_L and C_D, are calculated by fitting data points to a polynominal in factored form using a Lagrangian interpolation method.[9,10]

2. *Optimization Problem*

The problem is to find the control variable histories (T_p, T_a, α, i, δ_F) which minimize the annoyance function over the time interval $t \epsilon\ [t_o, t_f]$ subject to the equations of motion with specified initial conditions and the control bounds; $0 \leq T_P \leq (T_P)_{max}$; $0 \leq T_a \leq (T_a)_{max}$; $i_{min} \leq i \leq i_{max}$; and $\alpha_{min} \leq \alpha \leq \alpha_{max}$. Furthermore, any function of terminal values of the states or time t_f can be constrained. An inflight constraint is placed on the maximum allowable noise level at any listener as $PNL_i\ (Y_i, T) \leq 95$ PNdB. This constraint is a mixed function of the state and control variables.

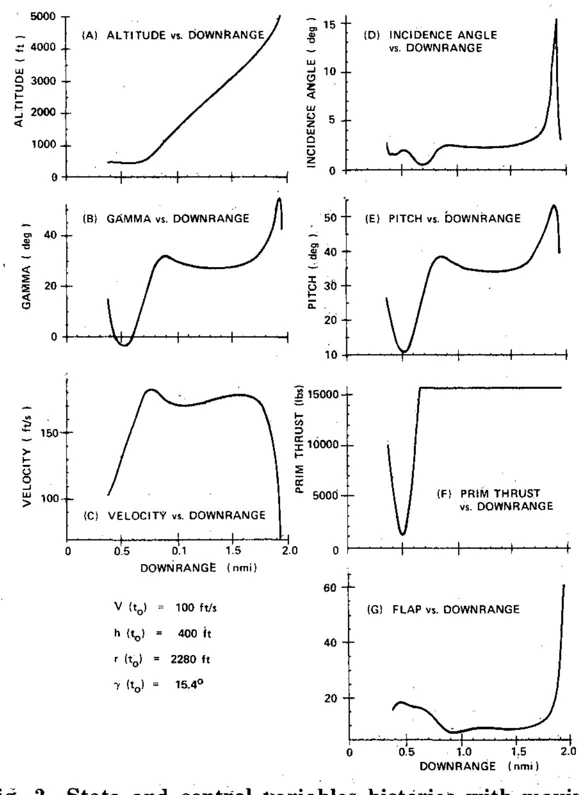

Fig. 2 State and control variables histories with maximum *PNL* constrained to 95 PNdB, T_{max} = 22,910 lb.

At first, angle of attack was used as a control variable. However, a phugoid motion developed which was difficult to control. Using pitch angle, θ, as the control variable, where $\theta = \gamma + \alpha$, the oscillation is eliminated. If constant θ command is used then as γ increases, α will decrease. By Eq. (7) γ is reduced so that the motion is stabilized.

Steepest descent[4] is a first-order gradient optimization algorithm that improves the performance index on each iteration. Included is this steepest descent study are inflight inequality constraints on the control variables and or state variables.[4,11] Constraint functions which are only functions of the control are included without any important programming changes. However, for constraint functions which explicitly contain the state variable and the control variable, additional programming is necessary because on the constraint boundary the control is a function of the state variable. Note that since pitch angle is the control variable, the angle of attack is a function of pitch angle and flight path angle. This angle of attack bound adds the same complexity as does the hard bound on PNL_i.

To minimize the annoyance function using the steepest descent formulation, each listener is represented by an additional state equation $g = 10^{(PNL_i/K)}$ so that Eq. (1) becomes

$$PI = \sum_{i=i}^{n} P_i S_i [K \log_{10}(g_i/t_o)]_{t=t_f} \tag{10}$$

The adjoint equations associated with the augmented states g_i are constants; thus, the addition of more listeners is relatively simple and not costly in computer time. The performance index can also be updated without much difficulty as better noise models become available.

III. Results

This analysis examines some general behavior patterns of minimum noise takeoff trajectories for STOL aircraft. In this context, minimum noise trajectories are obtained from various initial conditions to a final altitude of 5000 ft.

In Fig. 2 the state and control histories are plotted for the minimum noise trajectory using a maximum total engine thrust of 22,910 lb and initial conditions on the state variables of h = 400 ft, V = 100 fps, γ = 15.4°, and R = 2280 ft. By modulating the pitch, angle of incidence, and flap angle as shown in Figs. 2D, E, and G, the vehicle settles into a state of maximum sustainable flight path angle (γ = 30°). In this steady state condition the pitch angle is about 35° and the angle of incidence is close to zero. However, in order to keep the maximum *PNL* constrained to 95PNdB, the primary thrust was radically decreased from 15,870 lb to 2000 lb as it passed over listener one. It was seen that this caused only a slight increase in overall annoyance in comparison with runs made without the maximum PNL constraint. The angle of attack is constrained to an upper bound of 15° during the transient phase. While noise sensitive tracks for conventional jet aircraft range from 4 to 8 miles, no significant noise (70 PNdB is about the daytime background noise level in a residential neighborhood) is detected beyond a range of 2 miles.

At the end of each trajectory, a flare maneuver occurs where the velocity goes to zero and the altitude rises sharply. This occurs because the terminal condition was chosen to be an altitude of 5000 ft to save computer time. The program uses this flare to obtain a slight decrease in the performance index. However, if the terminal altitude were to be increased, the length of the steady-state climb would also increase. Therefore, the flare maneuver can be ignored and the trajectory assumed to end at the 5000 ft altitude in the steady state.

Minimum noise trajectories for ground takeoff to 5000-ft terminal altitude were also generated for both the constrained noise and unconstrained noise cases. Again, maximum sustainable flight path angle is achieved in steady state.

However, due to the constraint on instantaneous annoyance, the maneuvers are quite severe and unrealistic. Large thrust excursions occur when the noise is constrained to 95 PNdb at both microphone one (0.5 naut mile downrange) and microphone two (1 naut mile downrange). In fact, if a microphone were placed 0.75 naut mile from the runway (in addition to the 0.5-naut mile and 1.0-naut mile locations), the instantaneous noise level constraint of 95 PNdB would be violated. This presents a case for concentrating more microphones at the beginning of the ground track so that if the noise level constraint can be satisfied it will be satisfied more uniformly along the ground track, and hopefully a smoother trajectory would be generated.

To compare the effect of microphone position, the unconstrained ground takeoff case was repeated with all microphones shifted 0.5 naut mile downrange. Steady-state conditions were the same in both cases, but the time to reach steady state was less when the first microphone was downrange 1.0 naut mile instead of 0.5 naut mile.

IV. Conclusions

A steepest descent optimization program is used to determine minimum annoyance takeoff and climbout trajectories for a STOL vehicle. Although the vehicle model and annoyance function is quite sophisticated compared with previous work, improvements may be made by including in the annoyance function directional noise and disturbing high frequency noise components. However, until better noise field prediction techniques are developed, the results here are felt to give important trends. From various initial conditions, the trajectory begins with an initial transient which quickly settles into a steady state of maximum sustained flight path angle at full thrust. In the case of bounded maximum noise, the thrust drops radically during the initial transient. These maneuvers are unrealistic and might be softened with a slight increase the noise bound. For some choices of initial conditions, the angle-of-attack bound may be reached. This and the noise bound, which are functionally dependent upon both the state and control variables, are handled with only a little additional complexity in the steepest descent problem.[5] Thus, a far more sophisticated model is investigated without the need of the computationally cumbersome dynamic programming.

References

[1]DeMaio D. A., "Optimum Noise Abatement Trajectories," M..I.T., Unpublished Memo, March 1971, Charles Stark Draper Laboratory, Cambridge, Mass.

[2]Erzberger, H. and Lee, H. Q., "Technique for Calculating Optimum Takeoff and Climbout Trajectories for Noise Abatement," TN D-5182, May 1969, NASA.

[3]Hays, A. P., "Noise Minimization of Helicopter Take-off and Climb-out Flight Paths Using Dynamic Programming," MIT, Cambridge, Mass. M. S. thesis, May 1971, Dept. of Aeronautics and Astronautics, M.I.T.

[4]Bryson, A. E., Jr. and Ho, Y. C., *Applied Optimal Control*, Blaisdell, Waltham, Mass. 1969.

[5]Sperry, W. C., "Aircraft Noise Evaluation," TR FAA-no-68-34, Sept. 1968, FAA Office of Noise Abatement.

[6]Lee, R. et al, "Procedures for Estimating the Effects of Design and Operational Characteristics of Jet Aircraft on Ground Noise," CP-1053, June 1963, NASA.

[7]Boeing Company, "Study and Development of Turbofan Nacelle Modifications to Minimize Fan-Compressor Noise Radiation," Vol. 1, NASA CR-1711, Jan. 1971.

[8]Kramer, J. J., "Quick Engine Program, Detailed Engine Designs," Paper #19 in Progress of NASA Research Relating to Noise Alleviation of Large Subsonic Jet Aircraft, NASA SP-189, 1968.

[9]Hildebrand, F. B., *Introduction to Numerical Analysis*, McGraw-Hill, New York, 1956, pp. 60–64.

[10]Zeldin, S., and Speyer, J., "Maximum Noise Abatement Trajectories," AIAA Paper 72-665, 1972, Boston, Mass.

[11]Denham, W. F. and Bryson, A. E., Jr., "The Solution of Optional Programming Problems with Inequality Constraints," *AIAA Journal*, Vol. 1, No. 11, Nov. 1965, pp. 2544–2500.

On the Inverse Calculation of the Mass Flow Parameter

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On the Inverse Calculation of the Mass Flow Parameter

THE mass flow parameters for one-dimensional internal flow are related to the Mach number as

$$\dot{m}_s = M\left[\frac{\gamma g}{R}\left(1 + \frac{\gamma - 1}{2}M^2\right)\right]^{1/2} \quad (1)$$

$$\dot{m}_t = M\left[\frac{\gamma g}{R}\left(1 + \frac{\gamma - 1}{2}M^2\right)^{-\frac{\gamma+1}{\gamma-1}}\right]^{1/2} \quad (2)$$

where g is gravitation acceleration, R is the universal gas constant, and γ is the specific heat ratio. The mass flow parameters in terms of static and total pressures respectively are given as

$$\dot{m}_s = [W(T_t)^{1/2}/P_s A]; \qquad \dot{m}_t = [W(T_t)^{1/2}/P_t A]$$

Here W = weight flow rate, T_t = total temperature, P_s,P_t = static and total pressures respectively, and A = cross sectional flow area. The functional relationships between Mach number and m_s and m_t for air (γ = 1.4) are shown in Fig. 1.

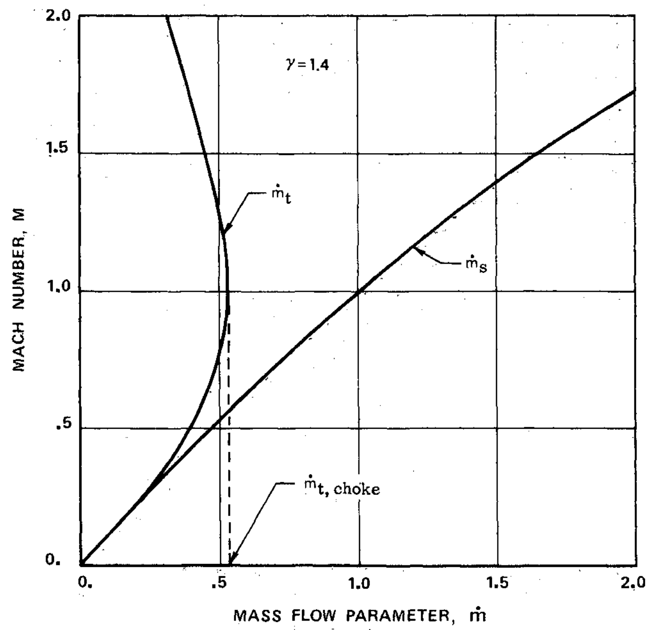

Fig. 1 Mach number variation with total pressure and static pressure mass flow parameters.

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